Lesson Research Proposal: Synthetic Geometry, Generalisation and Proof

Date of lesson: 5th March 2019
School name: St. Gerald's College
Teacher giving lesson: Mr. Joe Geraghty
Associate: Ms. Lynn Anderson
Lesson developed by: Ms. Louise Kelly, Ms. Mairéad Quinn, Mr. Gearoid O'Suilleabhean, Ms. Maggie Tighe
Lesson Plan Reviewers: Ms. Sinead Mahon and Ms. Julie Ryan

Title of the Lesson: Thinking outside the triangle.

Brief description of the lesson:

This lesson is developed to look at different approaches taken to the formalisation of proof with first years. Students will be asked using an investigative group task to analyse relationships. These facts together will be used to develop a formalised proof for the exterior angle and is intended to deepen their understanding of theorems in their studies.

Research Theme:

The theme of the lesson is the improvement of students' learning and encouraging them to analyse information. As the teachers are from a combination of schools, the group decided to focus on what they, as teachers, would like to see change with respect to the learning environment. Alongside this they focused on how teachers can improve the students' learning and communicative skills. Through Maslow's Hierarchy of Needs, we know that a safe and encouraging classroom environment is vital to create a positive learning experience. The role of the teacher is key here as they create the environment where students feel safe and able to communicate ideas without fear of ridicule. This makes one of our focuses creating a positive learning environment through the teacher's actions.

> "Teachers create an inclusive, orderly, student-centred learning environment based on mutual respect, affirmation and trust, in which students regulate and monitor their own behaviour" (Looking at our Schools, 2016).

A sub-theme that the group wished to be embedded in the lesson is the development of the students' communicative skills and responsibility for their own learning.

> "Students are motivated to learn through having a clear sense of attainable and challenging learning outcomes…They are able to work both independently and collaboratively in a very purposeful and productive manner" (Looking at our Schools 2016; School Self-Evaluation guidelines (SSE 2016 – 2020)

As part of our chosen theme, it was decided to incorporate group work into the process of this lesson to ensure all students actively engage with each other and are provided the opportunity to develop their communication skills.

Background & Rationale:

After a rich discussion encompassing how students learn and what they find difficult, the group was drawn to synthetic geometry. This is a topic where students are expected to become familiar with expressing orally their reasoning behind their answers. The emphasis for many will shift from the answer to justification of the answer. This is an important skill for problem-solving in the classroom and in developing a student's ability to make decisions. This is a key component for students' skill-set as they prepare for their classroom-based assessments.

Questions that arose from the discussion around Geometry were:
Why do students have a problem with expressing their ideas mathematically?
Why don't they have a conceptual understanding?
With a state exam corrector present as part of the team, they were able to highlight that from their experience the students had a good grasp of basic concepts but once deeper understanding and analysis were required in questions, most students fell short. Most of the group concurred that this was an issue not just in the examination process but in the classroom. Perhaps a different approach to the proofs would allow for greater understanding. It was felt that the first year of the implementation of the new Junior Cycle Specification could create the perfect opportunity for this group to investigate the value of generalisation and proof as part of the first-year units of learning.

Relationship of the Unit to the Syllabus:

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
Primary School: Students will have looked at the properties of shape	Throughout this unit, students will develop skills within the Learning	Draw on the knowledge/proofs that they have developed to solve

and on occasion used classification based on the properties Students should be able to estimate, measure and construct angles in degrees Students should be able to construct triangles from given sides or angles Students should be able to classify 2D shapes from their properties, also use angle and line properties to classify and describe angles and relate them to shape Explore the idea that the sum of three angles add to 180 degrees in a triangle, and explore the number of degrees in a quadrilateral	Outcomes: GT3 Investigate the concept of proof through their engagement with geometry so that they can • Recall and use the concepts of Axioms 1,2,3,5 • Build on their knowledge from theorems 1,3 and 5 • Investigate how relationship can lead to generalisation and proof U1, U2, U4, U11, U12, U13 Building Blocks Representation Generalisation and Proof Communication AF4a Select and use suitable strategies to find solutions to linear equations in a geometrical context	appropriate problems Using the skill of decomposing, identify the appropriate theorems needed to solve problems The communication skill will be drawn upon as students explore other possible problems and rigorously communicate their ideas and reasoning in 2nd and 3rd year.

Goals of the Unit:

The goal of this unit is to allow students the opportunity to develop their understanding around Geometry and expand their knowledge. They are also expected to advance their ability to communicate their ideas clearly, leading to the development of their understanding of the concept of proof. For this to happen in a meaningful way, students will be given the opportunity to draw upon prior knowledge and build on this through further investigation and reinforcement. As students take these opportunities, they will have to use their key skills to assist them in their progress.

Unit Plan

Alongside the development of students' knowledge and skills in relation to GT3, students will extend their knowledge of GT2, numbers, areas and aspects of the unifying strand.

Lesson	**Unit: Shape and Proof**
1	Polygons: Properties of shapes: Link to Primary School: Check in with students
2	Triangles: Properties of the shape
3	Sum of the angles in a triangle sum to 180 degrees
4	Research Lesson: Thinking outside the triangle!!
5	Each exterior angle of a triangle is equal to the sum of the opposite interiors angles
6	Isosceles triangles: Connection between angles and sides
7	Quadrilaterals and the importance of the triangle

Goals of the Research Lesson:

Mathematical Goals:

- Students will be given the opportunity to mathematically discuss a diagram. They are expected to write statements and provide justification/reasons for these statements.
- Students will be asked to analyse the facts they know and draw comparisons. The teacher will guide them through discussion, which is intended to lead the students to new learning.
- Students will be encouraged to appreciate the value of this theorem and asked to identify where else it can be applied, enabling them to further critique the diagram they have been given based on new knowledge.

Engagement with Statements of Learning:

During this lesson, students will be given the opportunity to:

- Organise, consolidate and communicate mathematical thinking clearly.
- To develop their ability to explore and understand patterns and relationships in a mathematical context (both contextual and abstract).

Key Skills:

As part of the process involved in lesson study, the group looked at the key skills that would be activated. After discussion, it was agreed to make the skills below the focus of our lesson.

Being Literate:

Students explain their thinking and justify their reasoning, using mathematical terminology appropriately and accurately.

Being Confident:

Students enjoy frequent opportunities to experience success in mathematics. Students will experience a positive approach to learning in which different approaches are valued and they are encouraged to learn from mistakes.

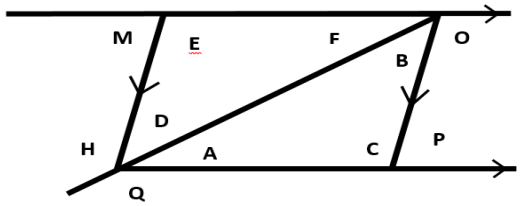
Learning with Others:

Students work together on collaborative tasks with peers where they develop both their mathematical and their interpersonal skills, offering mutual support and feedback throughout the process.

Wellbeing:

As previously mentioned in the theme, the group wanted to develop opportunities for a positive learning environment which can often be difficult when dealing with abstract concepts in Mathematics. The proposal is designed with this as a key theme. As a result, we should see students develop their positive disposition towards Mathematics and improve their awareness of others in this rich task.

Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction: (3 - 5min) Students will be settled into class and given the task. Teacher will highlight that there are two sets of parallel lines.		
Posing the Task: (3min) List all the connections that you can identify in the diagram.  See Appendix 1		

Student Group Work (10 min)

Students work in pairs to identify the relationships and together give their explanation for this relationship.

Straight line angles

$\|< m\| + \|< e\| = 180°$	sa
$\|< c\| + \|< p\| = 180°$	sa
$\|< h\| + \|< d\| = 180°$	sa
$\|< a\| + \|< q\| = 180°$	sa
$\|< o\| + \|< b\| + \|< f\| = 180°$	sa

(sa = straight angle)

Angles in a triangle

$\|< a\| + \|< b\| + \|< c\| = 180°$	aint
$\|< d\| + \|< e\| + \|< f\| = 180°$	aint

(aint = angles in a triangle)

Equal angles

$\|< o\| = \|< e\|$	ca
$\|< p\| = \|< d\| + \|< a\|$	ca
$\|< m\| = \|< f\| + \|< b\|$	ca
$\|< a\| = \|< f\|$	aa
$\|< b\| = \|< d\|$	aa
$\|< o\| = \|< c\|$	aa
$\|< p\| = \|< f\| + \|< b\|$	aa
$\|< m\| = \|< d\| + \|< a\|$	aa

(ca = corresponding angles)
(aa = alternate angles)

Angles at a point

$\|< a\| + \|< d\| + \|< h\| + \|< q\| = 360°$	angles around a point
$\|< e\| = \|< c\|$	opposite angles in a parallelogram
$\|< f\| + \|< b\| = \|< a\| + \|< d\|$	opposite angles in a parallelogram
$\|< e\| + \|< f\| + \|< b\| + \|< c\| + \|< a\| + \|< d\| = 360°$	total angles in a quadrilateral

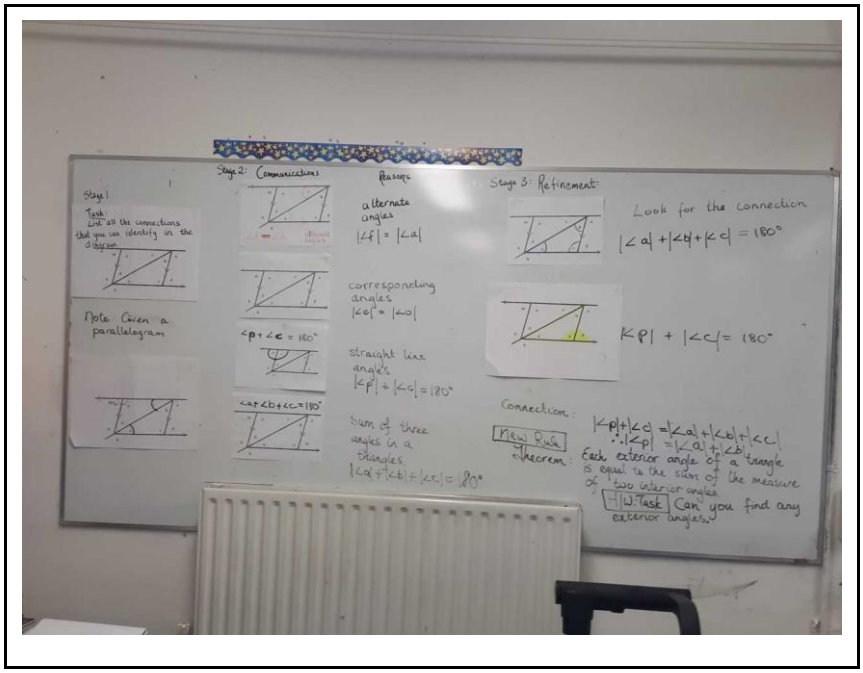
Equal in a triangle

$\|< m\| = \|< d\| + \|< f\|$	exterior angle
$\|< p\| = \|< a\| + \|< b\|$	exterior angle
$\|< q\| = \|< b\| + \|< c\|$	exterior angle
$\|< h\| = \|< e\| + \|< f\|$	exterior angle
$\|< p\| = \|< m\|$	

Ceardaíocht /Comparing and Discussing (12-15 mins) **As the student's work has been gathered, the teacher will ask four students to come forward and share their groups' findings.**		
Students will be asked to explain their answers in detail with an emphasis on their communication. 1. Alternate angles Any alternate angle statement 2. Corresponding angles Any corresponding angle statement 3. Straight Line angles $\|< p\| + \|< c\| = 180°$ 4. Sum of three angles $\|< a\| + \|< b\| + \|< c\| = 180°$	*Each student will explain their connection and provide the reason* *Four specific statements will be written on the board and from that the teacher will focus on the statements in 3 and 4.*	

The teacher will then take the statements from point 3 and 4, rewrite them and ask the students to focus on them for the next task. The students will be given time in their groups to see if they can find a **new relationship.** After three minutes the teacher will ask students to share their ideas and link their relationship statement to the exterior angle theorem (**NEW RULE**) See board plan	*Leading into the drawing of their knowledge, the teacher has specifically asked the students to list and explain point 3 and 4. Using the angles* $\lvert< p\rvert$, $\lvert< c\rvert$, $\lvert< a\rvert$, $\lvert< b\rvert$ *and* $\lvert< c\rvert$. *Focusing on what we have identified as key relationships. Can you come up with a new relationship in your groups?*	
Summing up & Reflection (4 mins)	**New Learning: Theorem** This will then lead the students to seeing the exterior angle rule and this will be introduced as a theorem **H/W task:** Find any further exterior angles relationships in the diagram.	

Board Plan



Evaluation Plan

The evaluation process was carefully planned, two observing teachers were observing the whole classroom engagement, while the remaining teachers were allocated a group of students each. The focus of this group of observers was to assess how students engaged with the task, each other and to what extent they engaged with the goals of the lesson.

Teachers assessing students work were recording comments alongside photographic evidence to support their observations.

Questions that the group were looking for the answers to were:

1. Did the majority of students achieve the goals of the lesson?
2. What were the main misconceptions?
3. What progress did the students make?
4. How did the lesson reflect the research theme?
5. Were the goals of the unit achieved?
6. Were the mathematical goals achieved?
7. Did the students engage with the statements of learning during the lesson?
8. What keys skills were activated during the lesson?
9. Did the teacher make any unplanned interventions?

Reflection based on the Lesson and Post-Lesson Discussion

During the post-lesson discussion, we reflected on the lesson and the value that all participants gained from the experience. The teacher who delivered the lesson was happy with how the lesson had gone. He felt that the students all had an opportunity to contribute to the class and was pleased that four students were able to achieve the new learning with little guidance. It was accepted that more guidance would be needed for other students. With regard to the homework task, it was agreed that this task could have been slightly more tailored to allow the weaker students access to another challenging question. The feedback from the observing teachers was that this lesson displayed a significant number of highly effective teaching practices. The teacher let students embrace the challenge of this task and offered support when required. It was introduced with a basic review of prior knowledge and then the task was given.

In the initial stages of this task, we were reminded that this is a concept that students of this age struggle with. The teacher was able to open the discussion up and draw upon their knowledge to clarify any issues enabling them to move forward. There was a significant change observed in the written statements from the early moments to the end of the task period. Initially students were writing angles down with no statement or connection to each other, however, as they discussed the task in their groups, these students transitioned to forming statements and expressing that angles were equal. Many students provided the reason for their connections.

The observing teachers were reminded of the value of time given to students. These students were able to decompose this diagram and gradually find connections successfully. They were all comfortably talking about the diagram and the connections. Their level of engagement during the Ceardaíocht was excellent and one teacher observed how the majority were eager to contribute. The teacher was also praised for acknowledging the students who successfully saw the new learning despite not being selected on this occasion to share with the group.

When critiquing the lesson proposal, we felt that it was clear that the research theme undoubtedly resonated throughout the lesson. Students worked independently and collaboratively to create new learning in a positive respectful environment. The goals of the unit were achieved as students developed their understanding and communicative skills. The mathematical goals were achieved to an extent; students discussed the diagram in detail. The teacher skilfully kept the group involved by checking statements. These comments included "*do you agree with this?*", "*what do you think of this statement*?". The group acknowledged that the

comparison of two specific statements proved very difficult for some students, but they all had displayed the skill of analysis in this task. The duration of the lesson did not allow for students to truly appreciate the value of this theorem and in hindsight this might have been more appropriate as a goal of the unit.

Our knowledgeable other shared with us a valuable statistic from her lesson note observations, through this entire lesson, the teacher spoke for just over 8 minutes of the lesson. The remaining time was composed of students on task, discussing or sharing their work. This information reiterates our key finding, time for our students to engage and process mathematical material is the most valuable learning they can receive.

Teacher Reflections on Lesson Study (some quotes):

"Meeting with fellow teachers and discussing the new specification, along with the curriculum for current junior cert has been an invaluable opportunity"

"My relationship with lesson study has developed my approach to questioning. Lesson study has given me the opportunity to develop how I look at questions and how I lead classroom discussions"

"Lesson study has enhanced my ability to change closed questions to open questions and increase the opportunity for deeper understanding"

"Working with maths teachers has widened my teaching strategies"

"Collaboration of teachers is invaluable, the best CPD that a teacher can do"

"students are given significant time to work on solutions. They get the opportunity to practise verbalising and presenting their answers to their peers"

"Exploring all connections and properties is a valuable opportunity to deepen the learning"

"Collaboration is a huge part of participation in Lesson study and the area where I benefit considerably."

"Well-being even just chatting to other teachers you realise they have the same difficulties with topics on the curriculum. "

"Over the years collecting a bank of excellent lessons to use with your own students. "

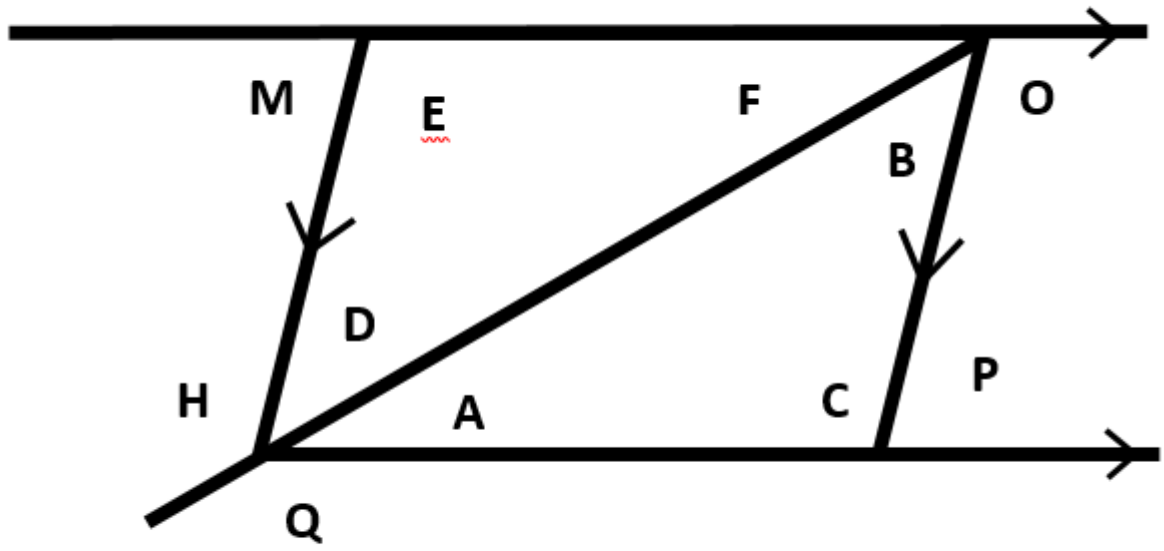
"Listening to ideas, looking at problems from a different perspective, learning from experienced teachers is of huge benefit."

"Being a reflective practitioner. How can I change, improve, try something new, adjust and learn by mistakes?"

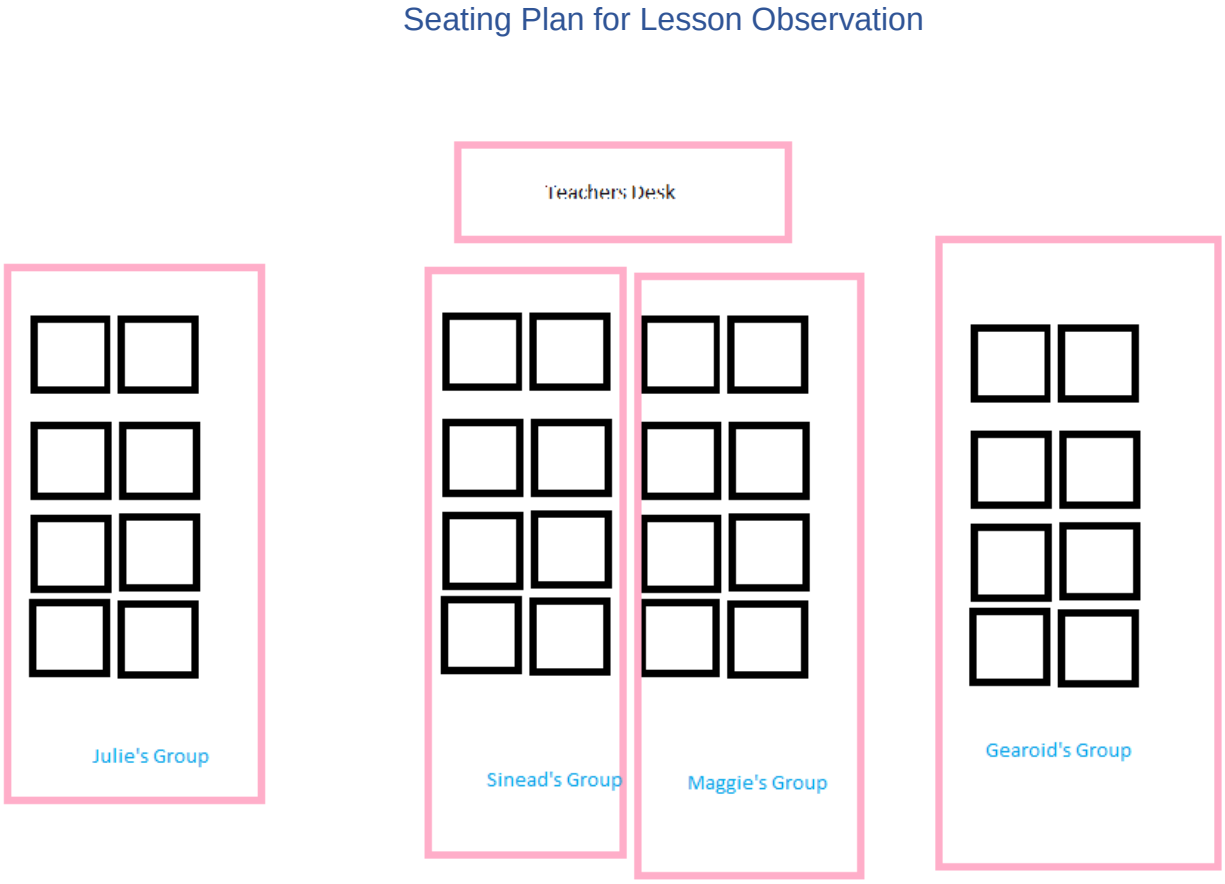
Appendix 1:

GROUP TASK:

List all the connections that you can identify in the diagram below.



Appendix 2: Seating Plan for Observation



Each teacher that is assigned a group will record some samples of students' work/ interesting comments from students/difficulties that students may encounter.

Lynn and Louise will observe the overall engagement of the class, interactions beyond groups, comments from students,